Basic Computer Math

Conversion of Numbers of Different Bases to Decimals

Consider the number 97531.

This is really: $9\times10000 + 7\times1000 + 5\times100 + 3\times10 + 1$.

This can also be written as: $9\times10^4 + 7\times10^3 + 5\times10^2 + 3\times10^1 + 1\times10^0$.

There is nothing special about why we use base 10 except that humans have ten fingers.

In the world of electronics, the natural base is two, because it only has two states: on and off. We use "1" to signify on, and "0" to signify off. This is referred to as the "binary system."

Example 1: Consider the binary number 1101011001.

To convert this to its decimal equivalent, we rewrite it as:
$1\times2^9 + 1\times2^8 + 0\times2^7 + 1\times2^6 + 0\times 2^5 + 1\times2^4 + 1\times2^3 + 0\times2^2 + 0\times2^1 + 1\times2^0$.

Or:
$1\times512 + 1\times 256 + 0\times128 + 1\times64 + 0\times32 + 1\times16 + 1\times8 + 0\times4 + 0\times2 + 1 = 857$.

Computational Devise "A"

Powers of 2	1024	512	256	128	64	32	16	8	4	2	1
Binary Digits		1	1	0	1	0	1	1	0	0	1
Their Product		512	256	0	64	0	16	8	0	0	1

Their sum: 512 + 256 + 0 + 64 + 0 + 16 + 8 + 0 + 0 + 1 = 857

Suppose we were to use another base, say "b." That would mean that the numbers representing the amounts multiplying their powers would range from "0" to one less that the base, i.e. "b-1." For example, in base ten, these numbers range from "0" to "9." That is why in base two we only have "0" and "1." The commonly used base in Information science is base sixteen, or hexadecimal ("hex" means six and "deci" refers to tenth).

Example 2: Consider the base four number $(1023)_4$. Notice the numbers range between "0" and "3," the subscript "4" indicates the base of the number.

$$1 \times 4^3 \quad 0 \times 4^2 + 2 \times 4^1 + 3 \times 4^0$$
$$1 \times 64 + 0 \times 16 + 2 \times 4 + 3 \times 1 = 73$$

Powers of 4	256	64	16	4	1
Base 4 Digits		1	0	2	3
Their Product		64	0	8	3

Adding the digits in the last row: 64 + 0 + 8 + 1 = 73

Example 3: $(647)_8$

$$6 \times 8^2 + 4 \times 8^1 + 7 \times 8^0$$
$$6 \times 64 + 4 \times 8 + 7 \times 1 = 415$$

Powers of 8	512	64	8	1
Octal Digits		6	4	7
Their Product		384	24	7

The sum of the digits in the last row 384 + 24 + 7 = 415

Notice that we have chosen bases that are powers of two. Those are the numbers that followed the progression within computer science. Because it is more convenient to use higher bases to represent quantities because it requires less space, or slots, to do so. The final base that the computational progression settled on was base sixteen or hexadecimal. Using base16, the range of numbers multiplying the powers of the base have to range between "0" and "fifteen." However, it was decided to represent the amounts from "ten" to "fifteen" by the letter A, B, C, D, E and F (i.e. A=10, B=11, C=12, D=13, C=14 and F=15).

Example 4: $(D3)_{16}$

$$D \times 16^1 + 3 \times 16^0$$
$$D \times 16 + 3 \times 1 = 13 \times 16 + 3 \times 1 = 209$$

Most of the hexadecimal numbers used in computer math will only have two places, thus representing numbers from 0 to FF (or 0 to 255).

Conversion of Decimals Numbers to Different Bases

Consider the number 97531.

Computational Devise "B"

Base Divisor	Dividend/Quotient	Remainder
10	97531	
10	9753	1
10	975	3
10	97	5
10	9	7
	0	9

Check:

Powers of 1	100000	10000	1000	100	10	1
Decimal Digits		9	7	5	3	1
Their Product		90000	7000	500	30	1

The sum of the digits in the last row is:

90000 + 7000 + 500 + 30 +1 = 97531

Notice how when you divide each dividend the remainder is the quantity that multiplies the base to the next lower power. And when you have a quotient of "0" you have finished, and the final remainder is the quantity that multiplies the highest powered base.

Example 5: Convert the decimal number 25 to its binary equivalent.

Base Divisor	Dividend/Quotient	Remainder
2	25	
2	12	1
2	6	0
2	3	0
2	1	1
	0	1

Check:

Powers of 2	1024	512	256	128	64	32	16	8	4	2	1
Binary Digits							1	1	0	1	1
Their Product							16	8	0	0	1

The sum of the digits in the last row 16×1 + 1×8 + 0×4 + 1×2 + 1×1 = 25

Example 6: $(48307)_{16}$

Base Divisor	Dividend/Quotient	Remainder
16	48307	
16	3019	3
16	188	11
16	11	12 = C
16	0	11 = B

Check:

Powers of 16	4096	256	16	1
Hexadecimal Digits	B = 11	C = 12	11	3
Their Product	45056	3072	176	3

Their sum 45056 + 3072 + 176 +3 = 48307

Hexadecimal Equivalents to the Decimal Numbers 0–255

Dec	Hex	Dec	Hex	Dec	Hex	Dec	Hex	Dec	Hex	Dec	Hex	Dec	Hex	Dec	Hex
0	0	32	20	64	40	96	60	128	80	160	A0	192	C0	224	E0
1	1	33	21	65	41	97	61	129	81	161	A1	193	C1	225	E1
2	2	34	22	66	42	98	62	130	82	162	A2	194	C2	226	E3
3	3	35	23	67	43	99	63	131	83	163	A3	195	C3	227	E3
4	4	36	24	68	44	100	64	132	84	164	A4	196	C4	228	E4
5	5	37	25	69	45	101	65	133	85	165	A5	197	C5	229	E5
6	6	38	26	70	46	102	66	134	86	166	A6	198	C6	230	E6
7	7	39	27	71	47	103	67	135	87	167	A7	199	C7	231	E7
8	8	40	28	72	48	104	68	136	88	168	A8	200	C8	232	E8
9	9	41	29	73	49	105	69	137	89	169	A9	201	C9	233	E9
10	A	42	2A	74	4A	106	6A	138	8A	170	AA	202	CA	234	EA
11	B	43	2B	75	4B	107	6B	139	8B	171	AB	203	CB	235	EB
12	C	44	2C	76	4C	108	6C	140	8C	172	AC	204	CC	236	EC
13	D	45	2D	77	4D	109	6D	141	8D	173	AD	205	CD	237	ED
14	E	46	2E	78	4E	110	6E	142	8E	174	AE	206	CE	238	EE
15	F	47	2F	79	4F	111	6F	143	8F	175	AF	207	CF	239	EF
16	10	48	30	80	50	112	70	144	90	176	B0	208	D0	240	F0
17	11	49	31	81	51	113	71	145	91	177	B1	209	D1	241	F1
18	12	50	32	82	52	114	72	146	92	178	B2	210	D2	242	F2
19	13	51	33	83	53	115	73	147	93	179	B3	211	D3	243	F3
20	14	52	34	84	54	116	74	148	94	180	B4	212	D4	244	F4
21	15	53	35	85	55	117	75	149	95	181	B5	213	D5	245	F5
22	16	54	36	86	56	118	76	150	96	182	B6	214	D6	246	F6
23	17	55	37	87	57	119	77	151	97	183	B7	215	D7	247	F7
24	18	56	38	88	58	120	78	152	98	184	B8	216	D8	248	F8
25	19	57	39	89	59	121	79	153	99	185	B9	217	D9	249	F9
26	1A	58	3A	90	5A	122	7A	154	9A	186	BA	218	DA	250	FA
27	1B	59	3B	91	5B	123	7B	155	9B	187	BB	219	DB	251	FB
28	1C	60	3C	92	5C	124	7C	156	9C	188	BC	220	DC	252	FC
29	1D	61	3D	93	5D	125	7D	157	9D	189	BD	221	DD	253	FD
30	1E	62	3E	94	5E	126	7E	158	9E	190	BE	222	DE	254	FE
31	1F	63	3F	95	5F	127	7F	159	9F	191	BF	223	DF	255	FF